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DEVELOPMENT OF PENMAN'S EQUATION FOR REGIONAL POTENTIAL TRANSPIRATION AND EVAPORATION

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Summary

The Penman-Brunt long wave radiation term, used in Penman's equation for estimating potential transpiration has been shown to under-estimate especially in the summer and transitional seasons. A simple method is developed for obtaining values of soil heat flux from changes of earth temperature. The same seasonal relationship was found when estimates of (Penman) potential evaporation were compared with measurements made with several irrigated lysimeters and with several black or unpainted British Standard evaporation tanks. 7-year average monthly values of the variables required by Penman's equation are used to provide comparisons between potential transpiration estimated by the original equation, by the equation as developed following Monteith, Thom and Oliver and including a seasonally adjusted long-wave radiation term and seasonal adjustment for net soil heat flux. Differences between sets of estimates of potential soil moisture deficit resulting from using 5 sets of estimates of potential transpiration are examined. It is shown that one version produces results which are supported by using empirical factors obtained from the analysis of evaporimeter relationships. Reference is made to encouraging results obtained from comparisons between estimated and neutron probe-derived measures of evaporation.

1. The long wave radiation tern in Penman's equation

Wales-Smith (1978) has compared monthly totals of $(1-r)R_c - R_N$ and monthly values of $\sigma T_a^4 (0.56-0.08 \sqrt{e_d})(0.10+0.90n/N)$, the former being indirect measurements of net, long-wave radiation, R_L, and the latter being estimates thereof, following Brunt (1939) and Penman (1948). The comparisons covered the period 1964-70 at Kew and Eskdalemuir Observatories and 1969-75 at Aldergrove.

R_c = measured (incoming) short-wave radiation

r = short-wave radiation albedo (taken as 0.25 for the three grassed sites)

R_N = measured net radiation (all wave-lengths)

σT_a^4 = black body radiation at screen temperature T_a °K

e_d = vapour pressure (on the screen) mb

n = average measured daily duration of bright sunshine

N = total possible average daily duration of bright sunshine for the latitude and time of year

He derived an empirical monthly factor, B, to adjust the Penman-Brunt estimates of R_L as follows

J	F	M	A	M	J	J	A	S	O	N	D
1.04	1.06	1.09	1.12	1.16	1.17	1.18	1.17	1.16	1.12	1.08	1.05

7-year averages of the measured and estimated radiation quantities at Kew and Eskdalemuir are given in Table 1. All four sets of averages values are worked up to give monthly averages of potential evaporation for grassland $E_{t\,p}$ following Penman (1948) in Table 2.

2. Soil Heat Flux

Penman (1948) referred to the partitioning of the heat budget at the evaporating surface including the change in stored heat. Bader and Wales-Smith (1977) have compared the soil heat flux (SHF_4) at 4cm depth with the change of temperature at 30cm depth (ΔT_{30}) at Cardington (Figure 1). It can be seen that, over the test periods, the two graphs were slightly out of phase and agree better if a 1-month lag is allowed; this is shown by the dots in Figure 1. ΔT_{30} is plotted against SHF_4, month-for-month in Figure 2 and with a 1-month lag in Figure 3; it can be seen that the scatter of points is appreciable reduced by introducing the lag.

A line of best fit, through the origin in Figure 3, gives the relationship $10\,\Delta T_{30}$(month n) = $3\,SHF_4$(month n+1) for T_{30} in °C and SHF_4 in equivalent mm of water evaporated (i.e. assuming total flux converted to latent heat of evaporation).

The average monthly change of temperature at 30 cm at Kew for the period 1931-60 (Meteorological Office, 1968) is plotted in Figure 4. Full arrows indicate net, monthly fluxes of heat to or from 30cm depth. Broken arrows indicate the sense and adjacent numbers indicate the magnitude of the assumed fluxes at 4cm on the basis of the empirical equation relating ΔT_{30} to SHF_4. It appears that in part of February although some extra heat is retained at 30cm it is not retained at 4cm, whilst in August there is a flux convergence somewhere between 30 and 4cm.

In Table 3 1964-70 average values of $\frac{\Delta}{\Delta+(1+r_{sdn}/r_a)\gamma}$, following Monteith (1965) as implied by Thom and Oliver (1977), in place of $\frac{\Delta}{\Delta+\gamma}$, to calculate the proportion of the surface energy balance used in providing the latent heat of evaporation, are multiplied by the monthly values of estimated SHF_4 given in Figure 4.

Δ = the slope of the saturation vapour pressure curve at a chosen temperature

γ = the coefficient of the wet and dry bulb hygrometer equation

r_a = the bulk aerodynamic resistance to evaporation (sm^{-1})

r_{sdn} = the bulk stomatal resistance to transpiration from freely transpiring dry vegetation (sm^{-1}).

The summer total of negative contribution to potential evapotranspiration is some 20mm which is reasonable in comparison with the corresponding contribution to actual evapotranspiration of 16mm quoted by Thom and Oliver (1977).

3. Relationships between Penman potential transpiration and evaporimeter evaporation/transpiration

With the aim of designing a method of quality-control for data from irrigated lysimeters (see Green, 1960) and evaporation tanks Wales-Smith (1975) compared calendar month estimated totals of $E_{t\,p}$ using an albedo value 0.25 with monthly totals of potential evapotranspiration obtained from six irrigated lysimeters (situated at Alice Holt Lodge, Cawood, Edgbaston, High Mowthorpe, Slaidburn and Sutton Bonington). The period of comparison totalled 53 station-years (with some missing totals). Monthly averages of the quantity (lysimeter evaporation/Penman evaporation: $E_{lys}/E_{t\,p}$ were calculated for each station. The seasonal variation of $E_{lys}/E_{t\,p}$ is in phase with the seasonal rise and fall of average temperatures at 30cm depth (Meteorological Office, 1968). Average values, $\overline{E_{LYS}}/\overline{E_{t\,p}}$, were obtained by fitting a smooth curve as closely as possible to the monthly median values.

The process was repeated using black or unpainted British Standard evaporation tank data from Edgbaston, Cawood, Wallingford, Cardington and Eskdalemuir and data from a black GG1 3000 tank at Kew (adjusted to give estimates of British Standard Tank evaporation using a factor obtained by Wales-Smith (1972).) An average curve of $\overline{E_{TANK}}/\overline{E_{t\,p}}$ for this 34 station years comparison was constructed by fitting a smooth curve to median values. The results are:

	J	F	M	A	M	J	J	A	S	O	N	D
$\overline{E_{LYS}}/\overline{E_{t\,p}}$	2.60	1.07	0.93	0.88	0.86	0.90	0.97	1.04	1.21	1.55	2.57	4.50
$\overline{E_{TANK}}/\overline{E_{t\,p}}$	2.20	1.07	0.91	0.86	0.86	0.89	0.95	1.01	1.15	1.55	4.30	6.60

4. Implications and support for modifications to Penman's equation for potential evaporation

Monthly average values of $\frac{\Delta}{\Delta+\gamma}$ and $\frac{\gamma}{\Delta+\gamma}$ for the period 1964-70 at Kew and Eskdalemuir are given in Table 2. Also given are average values of Penman's aerodynamic term, of the Penman-Brunt energy balance (net radiation) term and of measured net radiation. Monthly averages of Penman potential evaporation ($E_{t\,p}$) are obtained using both estimated and measured net radiation data.

Following Monteith (1965) Thom and Oliver (1977) introduced the quantity r_s (bulk actual stomatal resistance) into a version of Penman's equation, and also the quantity r_a (bulk aerodynamic resistance). To calculate potential transpiration the value of r_s used must be that appropriate to a given type of vegetation when dry and transpiring freely. This quantity, termed r_{sdn}, has been taken as $50sm^{-1}$ from November to February, $40sm^{-1}$ in March and October and $30sm^{-1}$ for the rest of the year. Average wind speeds at Kew and Eskdalemuir 1961-70 have been used to estimate

u_2 (wind speed at 2 metres) and r_a has been calculated using the expression $r_a = \left\{\ln z/zo\right\}^2 / K^2 u_2$ following Thom and Oliver 1977. Values of $r_{sd:n}/r_a$ have been worked out for Kew and Eskdalemuir in Table 3. z is taken as 2 metres, throughout but for Kew z (approximated as 1/10 of the vegetation height) has been taken as 0.02m, 0.01m and 0.005m (see Appendix). r_a has been calculated for Eskdalemuir for z_o = .02 only.

Wales-Smith (1975) found empirical relationships between $E_{t\,p}$ and E_{LYS}. These relationships were consistant with (a) under-estimation of winter potential evapotranspiration due to under-emphasis of the ventilation term in the combination equation, (b) under-estimation of winter E_t and over-estimation of summer E_t for lackofa seasonal net soil heat flux term and (c) over-estimation of transitional and summer season E_t due to under-estimation of net long-wave radiation.

Five monthly measures of E_t are compared (for Kew 1964-70) in Figure 5(a):

$E_{t\,p}$ = Penman's E_t for albedo 0.25

$E_{t\,p}\left\{\overline{E_{LYS}}/\overline{E_{tp}}\right\}$ = $E_{t\,p}$ adjusted by means of Wales-Smith's empirical factors

$E_{t(100)}$ = E_t (as already calculated in Tables 4(a), 4(b) and 4(c)
$E_{t(200)}$ = for z/z_o 100, 200 and 400. 100 probably represents
$E_{t(400)}$ = a typical English lowland catchment (Thom and Oliver (1977)) 200 and 400 may be appropriate to well-tended grass some 10 cm and 5 cm high.

The first three measures of E_t, only, were compared (for Eskdalemuir 1964-70) in Figure 5(b).

Considering Figure 5(a) it can be seen that the three versions of E_t are closer to the adjusted $E_{t\,p}$ than to original E_{tp} from October to January and that $E_{t\,200}$ and $E_{t\,400}$ are closer to adjusted $E_{t\,p}$ than is $E_{t\,100}$ from February to August.

Only $E_{t\,100}$ is shown for Eskdalemuir (Figure 5(b)) but it can be seen that lower values of E_t (ie for the higher values of z/z_o) would be closer to adjusted $E_{t\,p}$ than $E_{t\,100}$ for December to July.

These rough results must, however be interpreted with care. If the lysimeters were kept at field capacity and functioned correctly and if the measured rainfalls used in calculating individual values of E_{LYS} were representative of the falls upon the lysimeters the difference between $E_{t\,p}$ ($\overline{E_{LYS}}/\overline{E_{tp}}$) and E_t should be mainly due to the occurrence of evaporation at more than the potential transpiration rate following rainfall and irrigation. Climatological values of E_{LYS} would all be too large (in relation to potential transpiration) and hence values of E_{LYS}/E_{tp} would also be too large. On the other hand if the rainfall values used in computing individual values of E_{LYS} were, say, some 95% of the rain falling on the lysimeters (see Rodda, 1967, Clarkson, 1971 and Painter, 1975) the individual and climatological values of E_{LYS} (and hence $\overline{E_{LYS}}/\overline{E_{tp}}$ would all be too small by some 5% of the measured rainfalls.

5. Comparison of 5 measures of E_t via PSMD (potential soil moisture deficit)

The five measures of E_t shown in Figure 5 are used with average monthly rainfall to produce approximate estimates of average month-end PSMD for Kew for 1964-70 in Table 6. The values are plotted in Figure 6. Calculations were started in the first month in which the measure of E_t exceeded that of R (rainfall). It is interesting to note that PSMD based on E_{t100} does not return to zero and that the best agreement with PSMD based on $E_{t\,p}\left\{\overline{E_{LYS}}/\overline{E_{tp}}\right\}$ is given bythat based on E_{400}.

If as Thom, Oliver and others have suggested, $E_{t\,p}$ is a good approximation on an annual basis and if the rough, empirical factors obtained from the lysimeter comparison are approximately correct an annual total of about 556mm may be reasonable as an average for Kew for the period 1964-70 for a short grass site. The evaporation of intercepted water at more than the E_t rate is important especially where lysimeters are concerned and thus it is plausible that the correct potential transpiration total may be somewhat less than 556mm.

6. Comparison of potential and actual evapotranspiration estimates with data derived from neutron probe measurements

Wales-Smith and Arnott (1979) have made a number of comparisons of between estimated E_t and estimated , actual soil moisture deficit and measures of the same quantities derived from neutron probe data and have reported encouragingly good agreement.

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Figures and legends.

Figure 1 Comparison of monthly totals of soil heat flux at 4 cm and monthly change of temperature at 30 cm at Cardington 1966-67 and 1972-75.

Figure 2 Monthly totals of soil heat flux at 4 cm vs. monthly change of temperature at 30 cm (averages) at Cardington.

Figure 3 Totals of (average) monthly soil heat flux at 4 cm in month n+1 vs. change of temperature at 30 cm in month n.

Figure 4 Mean monthly change of earth temperature and sense of heat flux at 30 cm at Kew 1931-60 with assumed sense and magnitude of mean monthly soil heat flux at 4 cm. (30 cm solid arrows : 4 cm dashed arrows).

Figure 5 Mean monthly values of Penman's (1948) E_{tp} multiplied by average values of $\overline{E_{LYS}/E_{tp}}$, of Penman's E_{tp} and of $E_{t\ 100}$, $E_{t\ 200}$ and $E_{t\ 400}$ at Kew and Eskdalemuir ($E_{t\ 200}$ and $E_{t\ 400}$ not shown for Eskdalemuir). (1964-70)

Figure 6 Mean end-of-month values of potential soil moisture deficit at Kew for 1964-70 using E_{tp}, $E_{tp}(\overline{E_{LYS}/E_{tp}})$, $E_{t\ 100}$, $E_{t\ 200}$ and $E_{t\ 400}$.

Appendix.

$$r_a = \frac{\{\ln (z/z_0)\}^2}{K^2 u_z}$$

Let z = 2 m

Let h = vegetation height

= .2 m (20 cm)

Let $z_0 = h/10$

= .02

$\therefore z/z_0 = 2/.02 = 100$

$\{\ln 100\}^2 = 4.605^2 = 21.21$

Let z = 2m

Let h = .05

z_0 = .005

$\therefore z/z_0 = 2/.005 = 400$

$\{\ln 400\}^2 = 5.991^2 = 35.90$

Let z = 2m

Let h = .1 m (10cm)

z_0 = .01

z/z_0 = 200

$\{\ln 200\}^2 = 5.298^2 = 28.07$

$$m = \left[\frac{\ln (z/z_{0p})}{\ln (z/z_0)}\right]^2$$

z_{0p} = .00137 m

z = 2m

$\therefore z/z_{0p}$ = 1460

if $z/z_0 = 100$ $\quad m = \left[\frac{7.286}{4.605}\right]^2 = 2.50$

if $z/z_0 = 400$ $\quad m = \left[\frac{7.286}{5.991}\right]^2 = 1.48$

if $z/z_0 = 200$ $\quad m = \left[\frac{7.286}{5.298}\right]^2 = 1.89$

Table. 11.

Values of Measured and estimated radiation 1964-70

at Kew (units equiv. mm of water evaporated) assuming all energy so used.

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Net Radiation R_N	-10·5	-2·2	25·2	50·1	83·1	95·0	92·2	71·9	41·8	7·9	-13·1	-16·4	425·0
Global Rad^n. R_C	26·3	46·7	97·3	140·6	197·7	215·0	201·0	170·4	123·7	74·2	35·6	21·6	1350·1
Long wave Rad^n. R_L	-30·2	-37·2	-47·8	-55·4	-66·2	-66·3	-58·5	-55·9	-51·0	-47·8	-39·8	-32·6	-587·7
P. Net Rad^n. $R_{N(P)}$	-10·3	0·8	25·1	57·1	93·2	107·1	101·6	80·5	44·7	11·5	-9·6	-13·5	488·2
P. Global Rad^n. $R_{C(P)}$	28·0	47·7	97·8	142·0	196·7	213·0	199·1	173·3	123·8	74·0	36·6	22·7	1354·7
P. Long wave Rad^n $R_{L(P)}$	-31·3	-35·0	-48·2	-49·4	-54·3	-52·6	-47·7	-49·5	-48·2	-44·0	-37·1	-30·5	-527·8

Values of Measured and estimated radiation 1964-70

at Eskdalemuir (units are equiv. mm of water evaporated) assuming all energy so used

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Net Radiation R_N	-14·7	-9·2	19·5	54·3	77·6	101·9	84·1	70·5	34·5	6·4	-18·4	-27·1	379·4
Global Rad^n. R_C	19·6	48·7	88·6	134·3	166·4	199·8	173·3	152·6	97·7	57·8	29·4	17·1	1185·3
Long wave Rad^n R_L	-29·4	-45·7	-47·0	-46·4	-47·2	-45·0	-45·9	-44·0	-38·8	-36·9	-40·5	-39·9	-509·7
P. Net. Rad^n. $R_{N(P)}$	-15·6	-3·9	23·4	58·9	85·0	109·3	91·9	75·0	37·4	7·9	-16·4	-22·7	432·2
P. Global Rad^n. $R_{C(P)}$	20·0	44·3	86·1	136·8	166·0	202·7	170·5	149·7	94·9	58·6	29·6	17·6	1176·8
P. Long wave Rad^n $R_{L(P)}$	-30·6	-37·1	-41·2	-43·7	-39·5	-42·7	-36·0	-37·3	-33·8	-36·0	-36·6	-35·9	-450·4

Table 2

Values of Penman potential evaporation (albedo 0·25)

at Kew (1964-70)

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
$\Delta/\Delta+\gamma$	·478	·477	·501	·538	·595	·638	·654	·649	·623	·585	·515	·479	
$\gamma/\Delta+\gamma$	·522	·523	·499	·461	·404	·362	·345	·351	·376	·415	·485	·521	
E_{ap}	19·8	26·4	41·8	46·9	60·0	77·4	81·8	68·2	47·4	33·5	25·5	18·9	mm
$(\gamma/\Delta+\gamma)E_{ap}$	10·3	13·8	20·9	21·6	24·2	28·0	28·2	23·9	17·8	13·9	12·4	9·9	224·9 mm
$(\Delta/\Delta+\gamma)R_{N(P)}$	−4·9	0·4	12·6	30·7	55·4	68·3	66·5	52·3	27·9	6·7	−4·9	−6·5	304·5 mm
$(\Delta/\Delta+\gamma)R_N^*$	−5·0	−1·1	12·6	26·9	49·5	60·6	60·3	46·7	26·0	4·6	−6·7	−7·9	mm
$E_{t(P)}$	5·4	14·2	33·5	52·3	79·6	96·3	94·7	76·2	45·7	20·6	7·5	3·4	529·4 mm
$E_{t(P)}$ using R_N^*	(5·3)	(12·7)	(33·5)	(48·5)	(73·7)	(88·6)	(88·5)	(70·6)	(43·8)	(18·5)	(5·7)	(2·0)	(491·4) mm

Values of Penman potential evaporation (albedo 0·25)

at Eskdalemuir (1964-70)

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
$\Delta/\Delta+\gamma$	·437	·421	·451	·485	·536	·588	·590	·595	·573	·535	·461	·436	
$\gamma/\Delta+\gamma$	·563	·578	·549	·515	·464	·412	·410	·405	·427	·465	·539	·564	
E_{ap}	14·4	17·0	35·3	38·0	43·2	54·4	49·6	38·9	30·2	28·0	21·7	19·8	mm
$(\gamma/\Delta+\gamma)E_{ap}$	8·1	9·8	19·4	19·6	20·1	22·4	20·3	15·7	12·9	13·0	11·7	11·2	184·2 mm
$(\Delta/\Delta+\gamma)R_{N(P)}$	−6·8	−1·6	10·5	28·6	45·6	64·3	54·2	44·6	21·4	4·2	−6·6	−9·9	248·5 mm
$(\Delta+/\Delta+\gamma)R_N^*$	−6·4	−3·9	8·8	26·3	41·6	59·9	49·6	41·9	19·8	3·4	−8·5	−11·8	mm
$E_{t(P)}$	1·3	8·2	29·9	48·2	65·7	86·7	74·5	60·3	34·3	17·2	5·1	1·3	432·7 mm
$E_{t(P)}$ using R_N^*	(1·7)	(5·9)	(28·2)	(45·9)	(61·7)	(82·3)	(69·9)	(57·6)	(32·7)	(16·4)	(3·2)	(0·6)	(404·9) mm

TABLE 3.

Values of stages in the calculation of $r_{s\,min}/r_a$ - monthly averages - and of $\Delta+(1+r_{s\,min}/r_a)\gamma$ for Kew and Eskdalemuir 1964-70

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Kew U_2 ms^{-1}	3·2	3·6	3·6	3·3	3·1	2·8	2·6	2·6	2·5	2·5	3·1	3·1	
K^2u_2	·51	·58	·58	·53	·50	·45	·42	·42	·40	·40	·50	·50	for K=0·4
$z/z_0 = 2/.02 = 100$													
$\therefore \{\ln(z/z_0)\}^2 =$	21·21												
$\therefore \{\ln(z/z_0)\}^2/K^2u_2 = r_{a(100)}$	41·6	36·6	36·6	40·0	42·4	47·1	50·5	50·5	53·0	53·0	42·4	42·4	for $z/z_0=100$
$= r_{a(400)}$	70·4	62·1	62·1	67·9	72·0	80·0	85·7	85·7	90·0	90·0	72·0	72·0	for $z/z_0=400$
$= r_{a(200)}$	55·6	48·4	48·4	53·0	56·1	62·4	66·8	66·8	70·2	70·2	56·1	56·1	for $z/z_0=200$
$r_{s\,d(min)}$	50	50	40	30	30	30	30	30	30	40	50	50	
$r_{s\,dn}/r_{a(400)}$	·71	·81	·64	·44	·42	·37	·35	·35	·33	·44	·69	·69	for $z/z_0=400$
$r_{s\,dn}/r_{a(200)}$	·90	1·03	·83	·57	·53	·48	·45	·45	·43	·57	·89	·89	for $z/z_0=200$
$r_{s\,dn}/r_{a(100)}$	1·20	1·37	1·09	0·75	0·71	0·64	0·59	0·59	0·57	0·75	1·18	1·18	for $z/z_0=100$
1964-70 Kew Δ	·605	·602	·663	·770	·972	1·162	1·250	1·219	1·053	·929	·701	·606	
γ	·66	·66	·66	·66	·66	·66	·66	·66	·66	·66	·66	·66	
$\Delta+(1+r_{s\,dn}/r_{a\,400})\gamma$	1·73	1·80	1·75	1·72	1·91	2·07	2·14	2·11	1·97	1·88	1·82	1·72	m= 1·48
$\Delta+(1+r_{s\,dn}/r_{a\,200})\gamma$	1·86	1·94	1·87	1·81	1·98	2·14	2·21	2·18	2·04	1·97	1·95	1·85	m = 1·89
$\Delta+(1+r_{s\,dn}/r_{a\,100})\gamma$	2·05	2·18	2·05	1·89	2·09	2·22	2·31	2·28	2·15	2·05	2·15	2·06	m = 2·50
Eskdalemuir													
K^2u_2	·50	·57	·60	·51	·49	·46	·43	·42	·46	·51	·53	·51	
$\{\ln z/z_0\}^2 =$	21·21												
$\{\ln z/z_0\}^2/K^2u_2 = r_{a\,100}$	42·4	37·2	35·3	41·6	43·3	46·1	49·3	50·5	46·1	41·6	40·0	41·6	
$r_{s\,dn}$	50	50	40	30	30	30	30	30	30	40	50	50	
$r_{s\,dn}/r_{a\,100}$	1·18	1·34	1·13	0·72	0·65	0·65	0·61	0·59	0·65	0·96	1·25	1·20	
1964-70 Δ	·512	·481	·543	·622	·762	·941	·949	·871	·884	·758	·565	·510	
$\Delta+(1+r_{s\,dn}/r_{a\,100})\gamma$	1·95	2·03	1·95	1·76	1·88	2·03	2·01	2·02	1·97	2·05	2·05	1·96	

Authority and Remarks

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TABLE 4(a)

Values of $E_{t_{100}} = \frac{\Delta}{\Delta+(1+r_{sdm}/r_a)\gamma}\left\{(1-r)R_{cp} - .95BR_{LP} + G\right\} + \frac{2.58}{\Delta+(1+r_{sdm}/r_a)\gamma} E_{ap}$

at Kew 1964-70

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
$r_{sdm}/r_{2(100)}$	1·20	1·37	1·09	0·75	0·71	0·64	0·59	0·59	0·57	0·75	1·18	1·18	for $z/z_0 = 100$
$\Delta + (1+r_{sdm}/r_a)\gamma$	2·05	2·18	2·05	1·89	2·09	2·22	2·31	2·28	2·15	2·05	2·15	2·06	
$\Delta/\Delta+(1+r_{sdm}/r_a)\gamma$	·295	·278	·325	·400	·463	·518	·544	·537	·513	·446	·327	·296	
$(1-r)R_{cp}$	21·0	35·8	73·3	106·5	147·5	159·7	149·3	130·0	92·9	55·5	27·5	17·0	
$·95BR_{LP}$	31·0	35·3	49·7	52·4	58·7	58·4	53·4	54·9	53·0	46·6	38·2	30·5	
G	+6·0	+3·3	−1·3	−7·3	−10·3	−11·0	−9·3	−3·0	+3·7	+9·0	+11·2	+9·0	
$\{(1-r)R_{cp} - ·95BR_{LP} + G\}$	−4·0	3·8	22·3	46·8	77·5	90·3	86·6	72·1	43·6	17·9	0·5	−4·5	
$\frac{\Delta}{\Delta+(1+r_{sdm}/r_a)\gamma}\{\ \}$	−1·2	1·1	7·2	18·8	35·9	46·7	47·2	38·7	22·3	8·0	0·2	−1·3	
$2.58\gamma/\Delta+(1+r_{sdm}/r_a)\gamma$	0·805	·757	·805	·873	·789	·743	·714	·724	·767	·805	·767	·801	
E_{ap}	19·8	26·4	44·8	46·9	60·0	77·4	81·8	68·2	47·4	33·5	25·5	18·9	
$\left[\frac{2.58}{\Delta+(1+r_{sdm}/r_a)\gamma}\right] E_{ap}$	15·9	20·0	36·1	40·9	47·3	57·5	58·4	49·4	36·4	27·0	19·6	15·1	
$\frac{\Delta}{\Delta+(1+r_{sdm}/r_a)\gamma}\{\ \}+\frac{2.58}{\Delta+(1+r_{sdm}/r_a)\gamma}[E_{ap}]$	14·7	21·1	43·3	59·7	83·2	104·2	105·6	88·1	58·7	35·0	19·8	13·8	$= E_{t_{100}}$
E_{tp}	5·4	14·2	33·5	52·3	79·6	96·3	94·7	76·2	45·7	20·6	7·5	3·4	
$E_{t_{100}}/E_{tp}$	2·72	1·49	1·29	1·15	1·05	1·09	1·11	1·15	1·28	1·70	2·64	4·06	
Contribution to E_t due to G	+1·8	+0·9	−0·4	−3·0	−4·8	−5·7	−5·0	−1·6	+1·9	+4·1	+3·7	+2·7	
Contribution to E_t due to B	−0·4	−0·5	−1·3	−2·2	−3·8	−4·4	−4·4	−4·2	−3·7	−2·2	−0·9	−0·4	
$E_{T(P)}\{E_{tvs}/E_{tp}\}$	(14·0)	(15·2)	(31·1)	(46·0)	(68·5)	(86·7)	(91·9)	(79·3)	(55·3)	(31·9)	(20·6)	(15·3)	556
$\frac{\Delta+(1+r_{sdm}/r_{a_{100}})\gamma}{\Delta+\gamma}$	1·61	1·73	1·55	1·32	1·28	1·22	1·21	1·21	1·23	1·29	1·58	1·62	

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TABLE 4(b)

Values of $E_{t_{200}} = \frac{\Delta}{\Delta+(1+r_{sdm}/r_{a(200)})\gamma}\{(1-r)R_{cp}-.95BR_{LP}+G\} + \frac{1.98}{\Delta+(1+r_{sdm}/r_{a(200)})\gamma} E_{ap}$

at Kew 1964-70

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
$r_{sdm}/r_{a(200)}$	.90	1.03	.83	.57	.53	.48	.45	.45	.43	.57	.89	.89	for $z/z_0 = 200$
$\Delta+(1+r_{sdm}/r_a)\gamma$	1.86	1.94	1.87	1.81	1.98	2.14	2.21	2.18	2.04	1.97	1.95	1.85	
$\Delta/\Delta+(1+r_{sdm}/r_a)\gamma$	.325	.310	.355	.425	.491	.543	.566	.559	.536	.471	.359	.327	
$\{(1-r)R_{cp}-.95BR_{LP}+G\}$	-4.0	3.8	22.3	46.8	77.5	90.3	86.6	72.1	43.6	17.9	0.5	-4.5	
$\frac{\Delta}{\Delta+(1+r_{sdm}/r_a)\gamma}\{\ \}$	-1.3	1.2	7.9	19.9	38.1	49.0	49.0	40.3	23.4	8.4	0.2	-1.5	
$1.98/\Delta+(1+r_{sdm}/r_a)\gamma$	.674	.646	.671	.693	.633	.586	.567	.575	.615	.637	.643	.678	
E_{ap}	19.8	26.4	44.8	46.9	60.0	77.4	81.8	68.2	47.4	33.5	25.5	18.9	
$\frac{1.98}{\Delta+(1+r_{sdm}/r_a)\gamma} E_{ap}$	13.3	17.1	30.1	32.5	38.0	45.4	46.4	39.2	29.1	21.3	16.4	12.8	
$E_{t_{200}}$	12.0	18.3	38.0	52.4	76.1	94.4	95.4	79.5	52.5	29.7	16.6	11.3	
E_{tp}	5.4	14.2	33.5	52.3	79.6	96.3	94.7	76.2	45.7	20.6	7.5	3.4	
$E_{t_{200}}/E_{tp}$	2.22	1.29	1.13	1.00	.96	0.98	1.01	1.04	1.15	1.44	2.21	3.32	

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TABLE 4(c)

Values of $E_t = \frac{\Delta}{\Delta+(1+r_{sam}/r_a)\gamma_{400}}\{(1-r)R_{cp}-.95BR_{LP}+G\} + \frac{1.5}{\Delta+(1+r_{sam}/r_a)\gamma_{400}} E_{ap}$

at Kew 1964-70 (E_t is compared with Penman's $E_{t(P)}$)

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
$r_{sam}/r_{a(400)}$	.71	.81	.64	.44	.42	.37	.35	.35	.33	.44	.69	.69	for $z/z_0=400$
$\Delta+(1+r_{sam}/r_a)\gamma$	1.734	1.797	1.745	1.720	1.909	2.066	2.141	2.110	1.971	1.879	1.816	1.721	
$\Delta/\{\Delta+(1+r_{sam}/r_a)\gamma\}$	.349	.335	.380	.448	.509	.562	.584	.578	.555	.494	.386	.352	
$(1-r)R_{cp}$	21.0	35.8	73.3	106.5	147.5	159.7	149.3	130.0	92.9	55.5	27.5	17.0	
B	1.04	1.06	1.09	1.12	1.16	1.17	1.18	1.17	1.16	1.12	1.08	1.05	
.95B	.99	1.01	1.03	1.06	1.10	1.11	1.12	1.11	1.10	1.06	1.03	1.00	
R_{LP}	31.3	35.0	48.2	49.4	54.3	52.6	47.7	49.5	48.2	44.0	37.1	30.5	
$.95BR_{LP}$	31.0	35.3	49.7	52.4	59.7	58.4	53.4	54.9	53.0	46.6	38.2	30.5	
G	+6.0	+3.3	−1.3	−7.3	−10.3	−11.0	−9.3	−3.0	+3.7	+9.0	+11.2	+9.0	
$\{(1-r)R_{cp}-.95BR_{LP}+G\}$	−4.0	3.8	22.3	46.8	77.5	90.3	86.6	72.1	43.6	17.9	0.5	−4.5	
$\left[\frac{\Delta}{\Delta+(1+r_{sam}/r_a)\gamma}\right]\{(1-r)R_{cp}-.95BR_{LP}+G\}$	−1.4	1.3	8.5	21.0	39.5	50.7	50.6	41.7	24.2	8.8	0.2	−1.6	
$1.58/\{\Delta+(1+r_{sam}/r_a)\gamma\}$	.571	.551	.567	.576	.519	.479	.462	.469	.502	.527	.545	.575	for $z/z_0=400$
E_{ap}	19.8	26.4	44.8	46.9	60.0	77.4	81.8	68.2	47.4	33.5	25.5	18.9	
$\left[\frac{1.58}{\Delta+(1+r_{sam}/r_a)\gamma}\right] E_{ap}$	11.3	14.5	25.1	27.0	31.1	37.1	37.8	32.0	23.8	17.7	13.9	10.9	
$E_{t_{400}}$	9.9	15.8	33.6	48.0	70.6	87.8	88.4	73.7	48.0	26.5	14.1	9.3	
Penman's $E_{t(P)}$	5.4	14.2	33.5	52.3	79.6	96.3	94.7	76.2	45.7	20.6	7.5	3.4	
$E_{t_{400}}/E_{t(P)}$	1.83	1.11	1.00	0.92	0.89	0.91	0.93	0.97	1.05	1.29	1.88	2.73	

TABLE 5

Values of $E_{t_{100}} = \frac{\Delta}{\Delta + (1 + r_{s_{20m}}/r_{a_{100}})\gamma} \left\{ (1-r)R_{cp} - .95BR_{Lp} + G \right\} + \frac{2.5\delta}{\Delta + (1 + r_{s_{20m}}/r_{a_{100}})\gamma} E_{\partial p}$

at Eskdalemuir 1964-70.

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
$1 + r_{s_{20m}}/r_{a_{100}}$	1·18	1·34	1·13	0·72	0·69	0·65	0·61	0·59	0·65	0·96	1·25	1·20	$\frac{a}{20} = 100$
$\Delta + (1 + r_{s_{20m}}/r_{a_{100}})\gamma$	1·95	2·03	1·95	1·76	1·88	2·03	2·01	2·02	1·97	2·05	2·05	1·96	
$\Delta / \Delta + (1 + r_{s_{20m}}/r_{a_{100}})\gamma$	·263	·237	·278	·353	·405	·463	·472	·481	·449	·370	·276	·260	
$(1-r)R_{cp}$	15·0	33·2	64·6	102·6	124·5	152·0	127·9	112·3	71·2	43·9	22·2	13·2	
$·95BR_{Lp}$	30·2	37·4	42·7	46·5	43·5	47·5	40·4	41·5	37·3	38·3	37·5	35·8	
G	+6·0	+3·3	−1·3	−7·3	−10·3	−11·0	−9·3	−3·0	+3·7	+9·0	+11·2	+9·0	
$\{(1-r)R_{cp} - ·95BR_{Lp} + G\}$	−9·2	−0·9	20·6	48·8	70·7	93·5	78·2	67·8	37·6	14·6	−4·1	−13·6	
$\frac{\Delta}{\Delta + (1 + r_{s_{20m}}/r_a)\gamma} \{\ \}$	−2·4	−0·2	5·7	17·2	28·6	43·3	36·9	32·6	16·9	5·4	−1·1	−3·5	
$2.5\delta / \Delta + (1 + r_{s_{20m}}/r_a)\gamma$	·846	·813	·846	·937	·878	·813	·821	·817	·837	·805	·805	·842	
$E_{\partial p}$	14·4	17·0	35·3	38·0	43·2	54·4	49·6	38·9	30·2	28·0	21·7	19·8	
$\frac{2.5\delta}{\Delta + (1 + r_{s_{20m}}/r_a)\gamma} E_{\partial p}$	12·2	13·8	29·9	35·6	37·9	44·2	40·7	31·8	25·3	22·5	17·5	16·7	
$\frac{\Delta}{\Delta + (1 + r_{s_{20m}}/r_a)\gamma} \{\ \} + \frac{2.5\gamma}{\Delta + (1 + r_{s_{20m}}/r_a)\gamma} E_{\partial p}$	9·8	13·6	35·6	52·8	66·5	87·5	77·6	64·4	42·2	27·9	16·4	13·2	$= E_{t_{100}}$
E_{tp}	1·3	8·2	29·9	48·2	65·7	86·7	74·5	60·3	34·3	17·2	5·1	1·3	
$E_{t_{100}}/E_{tp}$	7·54	1·66	1·19	1·09	1·01	1·01	1·04	1·07	1·23	1·62	3·22	10·15	
$E_{t(p)} \left\{ E_{LVS}/E_{tp} \right\}$	(3·4)	(5·8)	(27·8)	(42·4)	(56·5)	(78·0)	(72·3)	(62·7)	(41·5)	(26·7)	(14·0)	(5·9)	
Δ	·512	·481	·543	·622	·762	·941	·949	·971	·884	·758	·565	·510	

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TABLE 6

Values of Potential Soil Moisture Deficit (PSMD)

at Kew 1964-70

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
$E_{t(P)}$	5	14	33	52	80	96	95	76	46	21	7	3	528
R	44	35	43	56	52	52	63	64	55	45	64	58	
PSMD					28	72	104	116	107	83	26	0	
$E_{t\,100}$	15	21	43	60	83	105	105	88	59	35	20	14	648
R	44	35	43	56	52	52	63	64	55	45	64	58	
PSMD	31	17	17	4	35	88	130	154	158	148	104	60	
$E_{t\,200}$	12	18	38	52	76	94	95	79	53	30	17	11	576
R	44	35	43	56	52	52	63	64	55	45	64	58	
PSMD	0				24	66	98	113	111	96	49	2	
$E_{t\,400}$	10	16	34	48	71	88	88	74	48	27	14	9	526
R	44	35	43	56	52	52	63	64	55	45	64	58	
					19	55	80	90	83	65	15	0	
$E_{t(P)}\{E_{LYS}/E_{LP}\}$	14	15	31	46	69	87	92	79	55	32	21	15	556
R	44	35	43	56	52	52	63	64	55	45	64	58	
PSMD					17	52	81	96	96	83	40	0	

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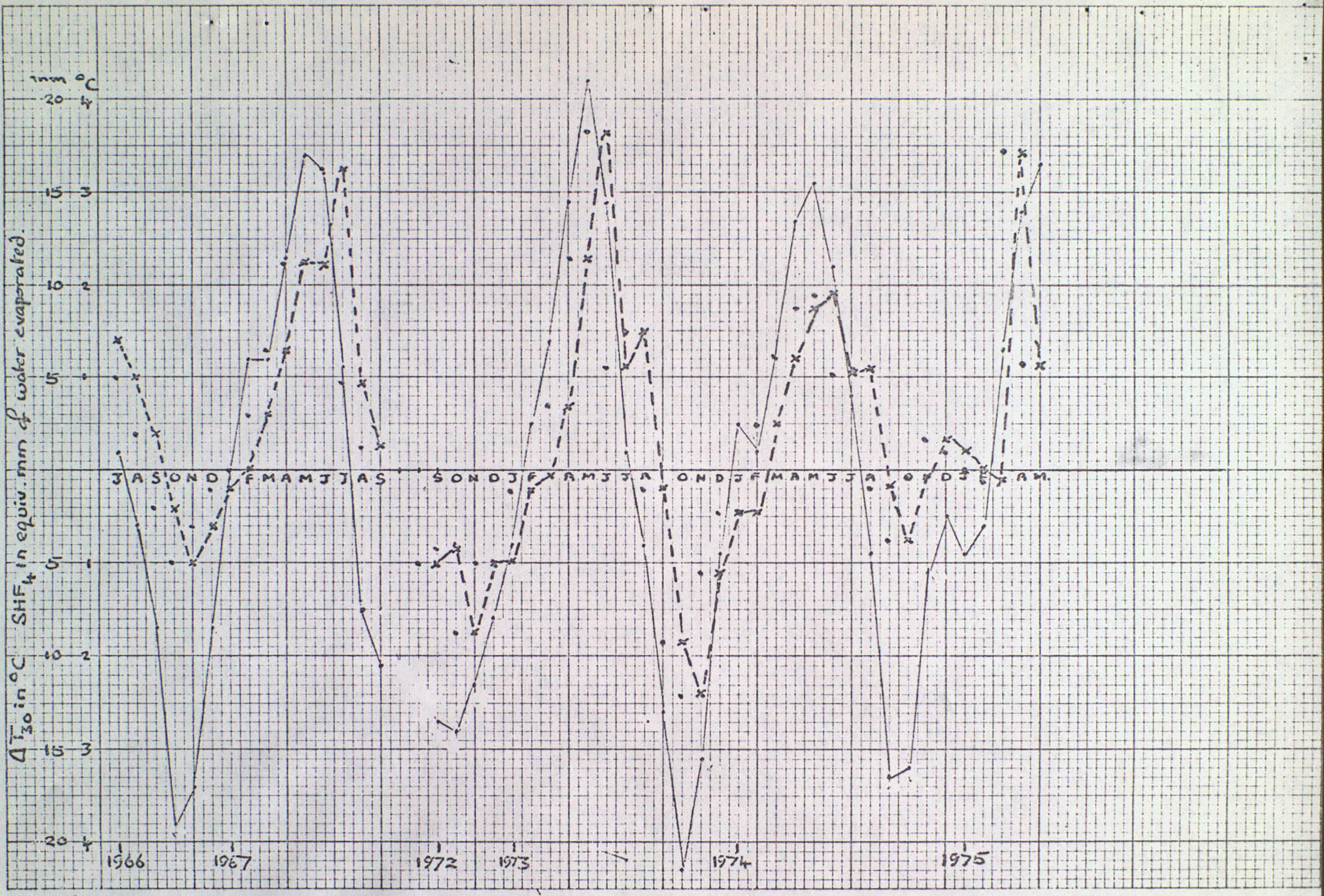
mm °C
20 4
15 3
10 2
5 1
10 2
15 3
20 4
ΔT_{30} in °C SHF$_4$ in equiv. mm of water evaporated.
J A S O N D J F M A M J J A S S O N D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M.
1966
1967
1972
1973
1974
1975

Fig. 9.1

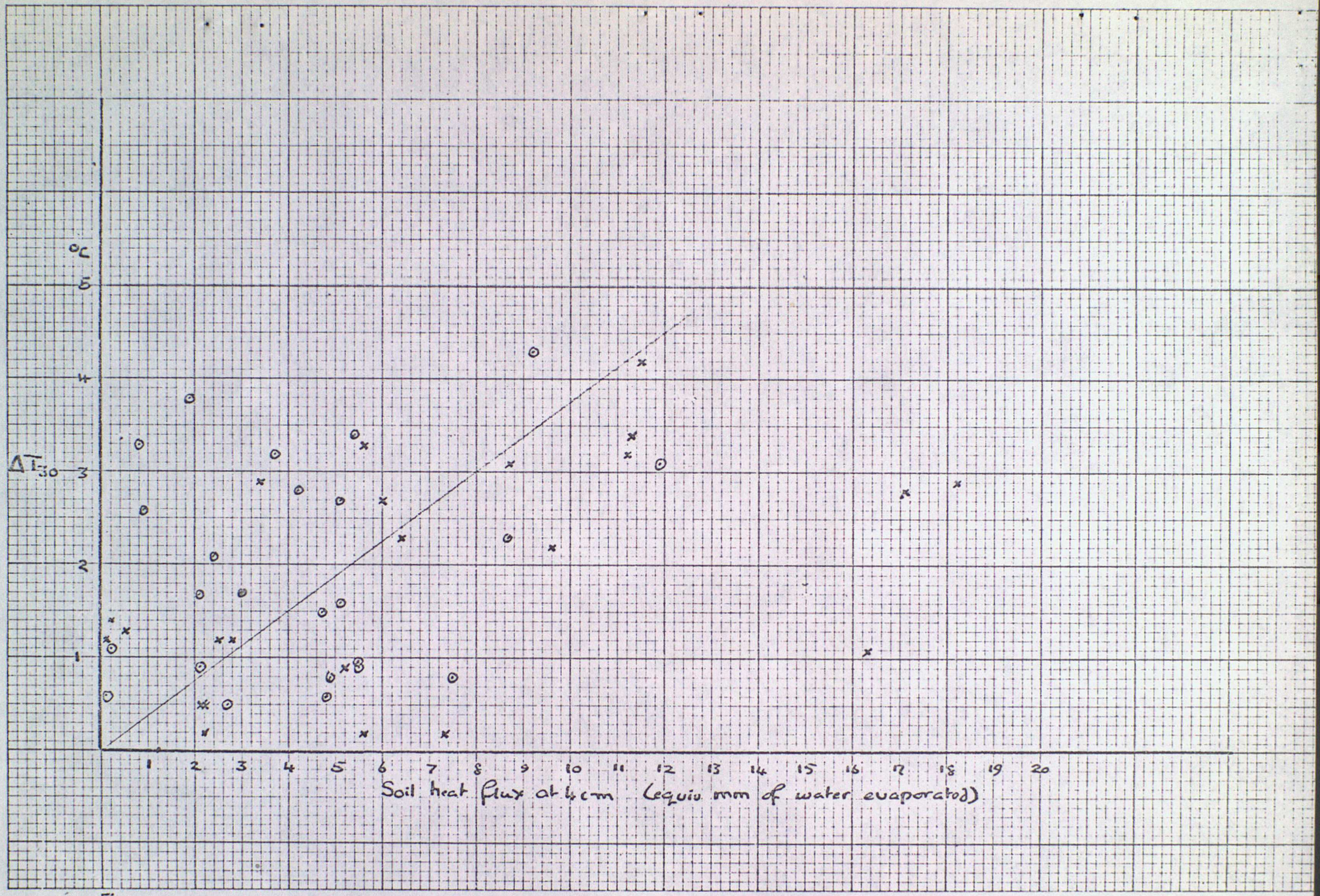
°C
5
4
3
2
1
ΔT_{30}
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Soil heat flux at 4 cm (equiv mm of water evaporated)

Fig. ~~10~~. 2

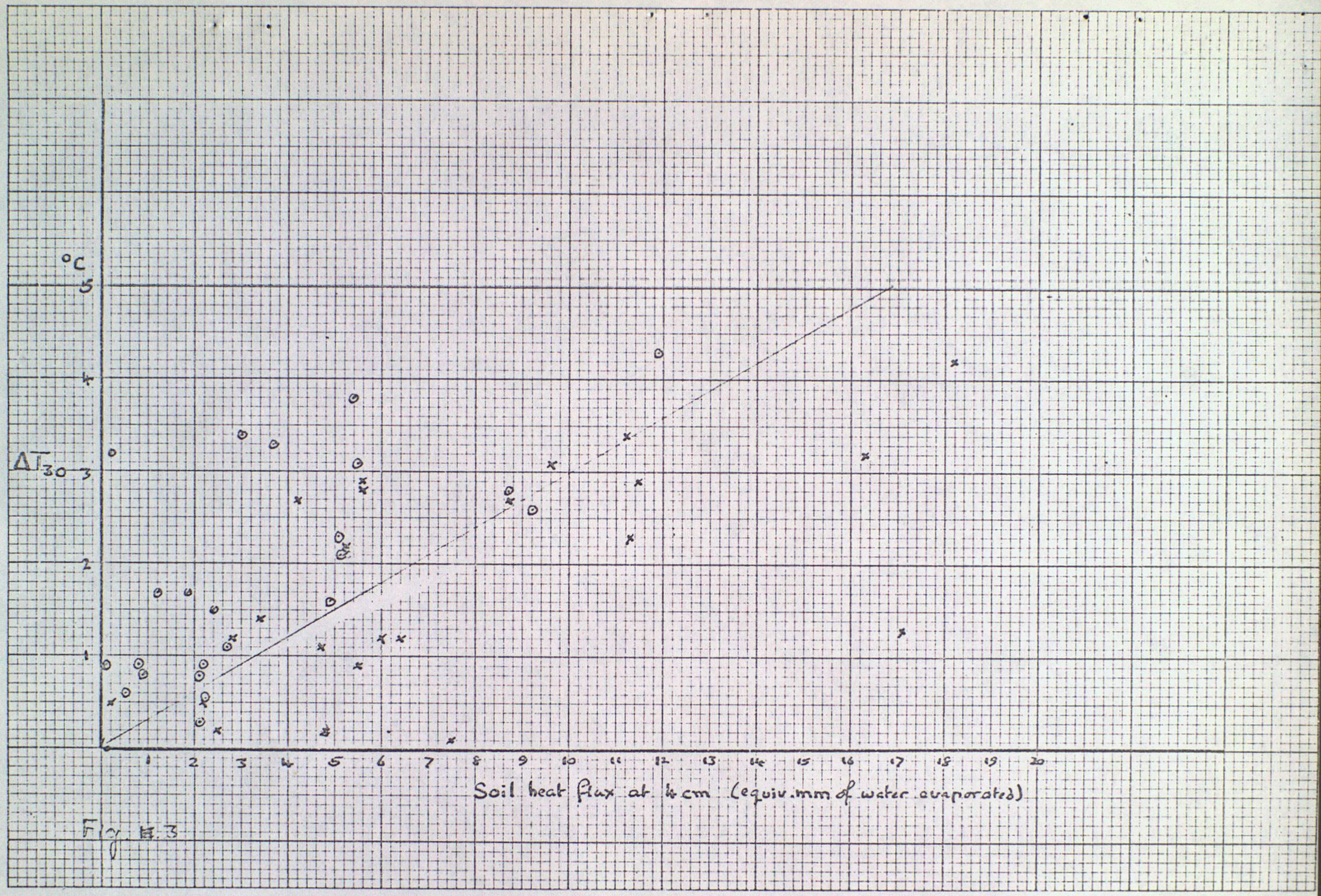
°C
5
4
3
2
1
$\Delta\overline{T}_{30}$
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Soil heat flux at 4 cm (equiv. mm of water evaporated)

Fig. 3

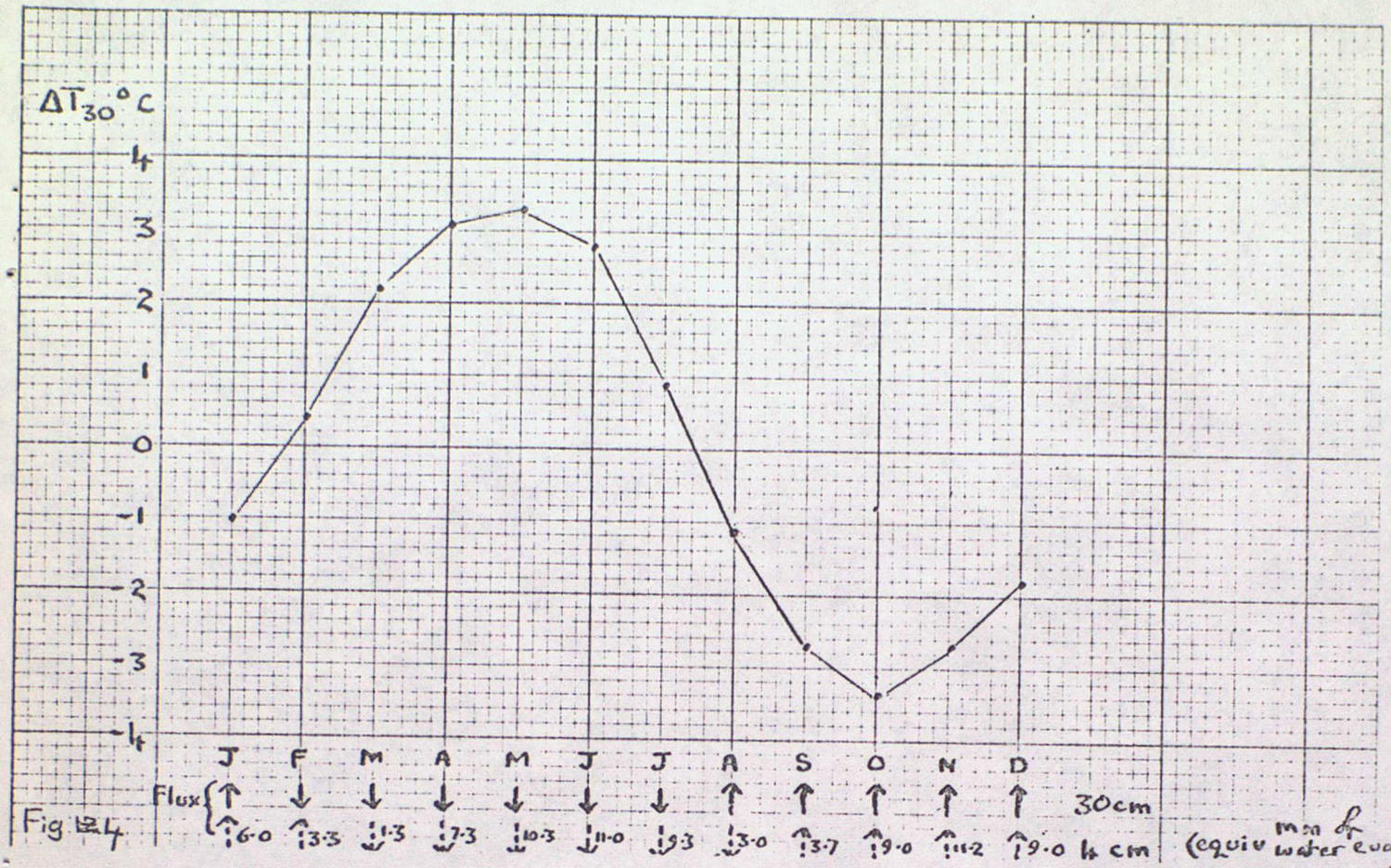
ΔT30 °C
4
3
2
1
0
-1
-2
-3
-4
J F M A M J J A S O N D
Flux
6·0 3·3 1·3 7·3 10·3 11·0 9·3 3·0 3·7 9·0 11·2 9·0 4 cm
30cm
(equiv mm of water evap)

Fig 24

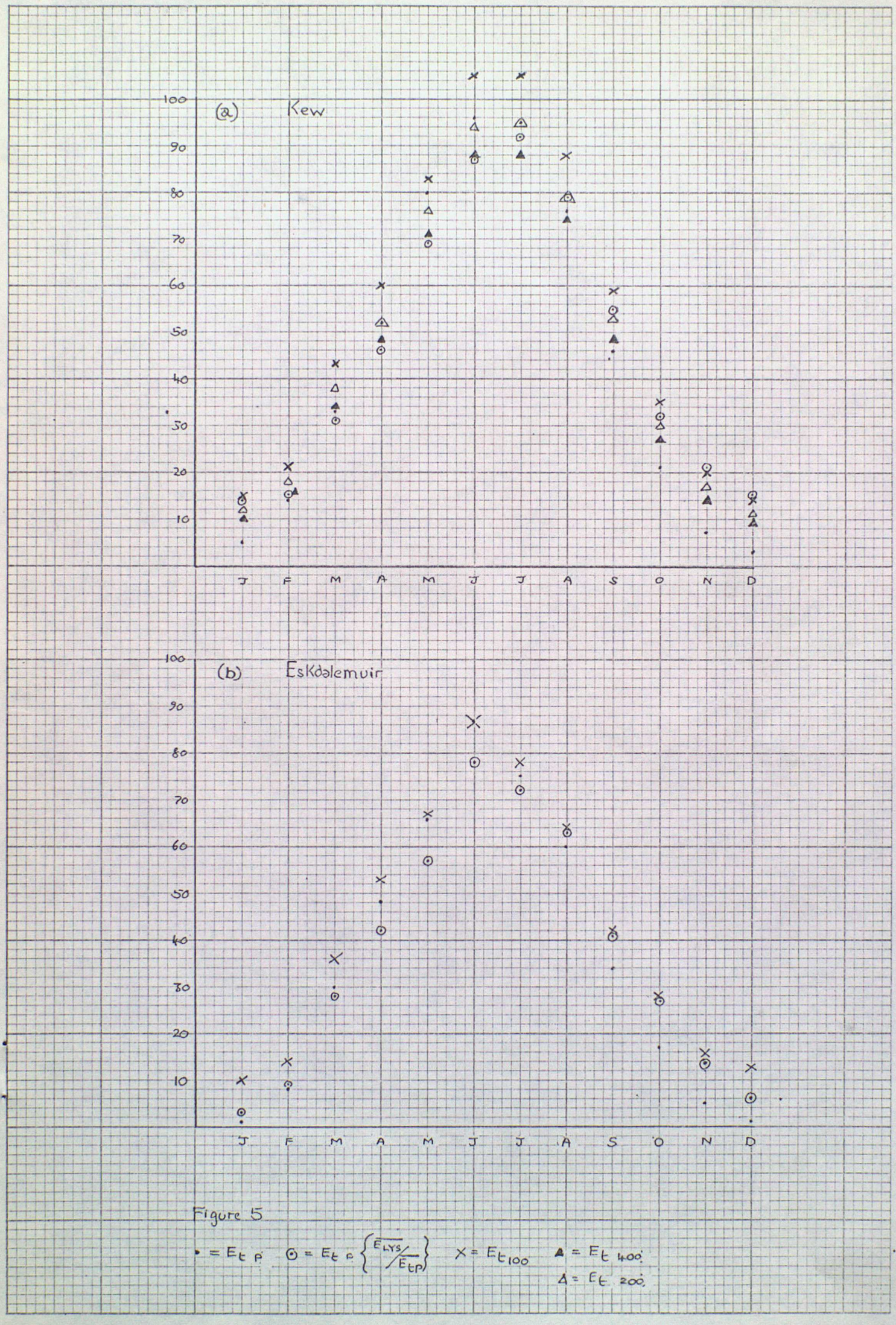

Figure 5

• = $E_{t\,P}$ ⊙ = $E_{t\,P}\left\{\frac{E_{LYS}}{E_{tP}}\right\}$ X = $E_{t\,100}$ ▲ = $E_{t\,400}$ △ = $E_{t\,200}$

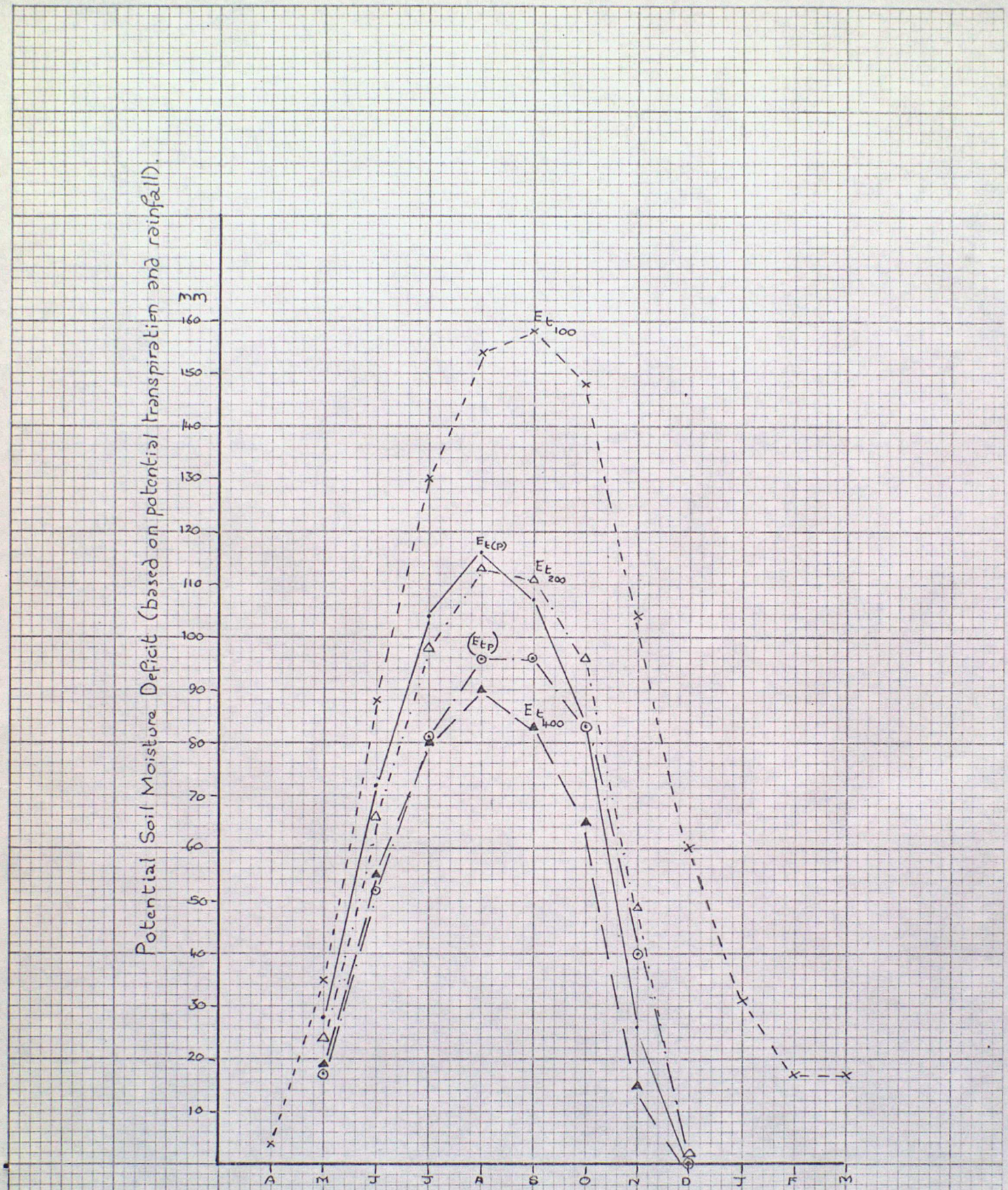
Potential Soil Moisture Deficit (based on potential transpiration and rainfall).
mm
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
A
M
J
J
A
S
O
N
D
J
F
M
Et100
Et(P)
Et200
(Et P)
Et400

Figure 6.